# Grade 4 Mathematics Instructional Unit 2: Segment 1

**Unit Big Idea:** Change  
**Segment Idea:** Changing from single digit to multi-digit

**Suggested Duration:** 15 Days

## What do we want all students to learn?

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>NBT.4</td>
<td>NS3.1*</td>
<td>Mathematical Practices</td>
<td>Mathematical Practices</td>
</tr>
<tr>
<td>NBT.5</td>
<td>NS3.2*</td>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>NBT.6</td>
<td>NS3.3*</td>
<td>7. Look for and make use of structure.</td>
<td>2. Reason abstractly and quantitatively</td>
</tr>
<tr>
<td></td>
<td>NS3.4*</td>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>5. Use appropriate tools strategically</td>
</tr>
</tbody>
</table>

**Target(s):**
E: Use place value understanding and properties of operations to perform multi-digit arithmetic.

## Skills and Concepts

**When learning Concepts and Procedures students will . . .**
- Fluently add and subtract multi-digit whole numbers using the standard algorithm. (NBT.4)
  - add multi-digit whole numbers.
  - subtract multi-digit whole numbers.
- Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (NBT.5)
  - multiply a whole number up to four digits by a one digit whole number.
  - multiply two two-digit numbers.

**When problem solving students will . . .**
- problem solve by adding, subtracting, multiplying and dividing multi-digit numbers.

**While modeling and analyzing data students will . . .**
- model their understanding of adding, subtracting, multiplying and dividing by demonstrating an understanding of base ten numeral concepts.

**While communicating and reasoning students will . . .**
- illustrate and explain a multiplication and division problem using the strategy of their choice.
• illustrate and explain multiplication by using equations, rectangular arrays, and/or area models.

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (NBT.6)

• divide a whole number up to four digits by a one digit whole number.
• recognize the relationship between multiplication and division.
• illustrate and explain division using equations, rectangular arrays, and/or area models.

<table>
<thead>
<tr>
<th>Language Functions and Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sequencing</strong> – Use properties of operations to add, subtract, multiply and divide multi-digit numbers through a series of steps. Domain specific vocabulary used.</td>
</tr>
<tr>
<td><strong>Organize</strong> – Using a pictorial representation and/or manipulatives, express relationships between/among structures and/or the arrangement of information. Domain specific vocabulary used.</td>
</tr>
<tr>
<td><strong>Argumentation</strong> – Phrases and sentences used to present a point of view to support/explain the process. Domain specific vocabulary used.</td>
</tr>
</tbody>
</table>

Language Considerations: simple/complex sentences, to be verbs (is/are), adverbials (because).

____ can be shown by using __________, because __________.

Language Considerations: simple sentences, to be verbs (is/are), conjunctions.

____ can be shown by using ____.

Language Considerations: simple sentences, to be verbs (is/are), dependent clauses.

The _______ of ______ is _____ because ______.
How will we know if they have learned it?

Performance Task and Formative Development:
In 15 Instructional days students will be able to........

Mastery of Concepts and Skills: 16/20

Selected Response:
Select three numbers that add to 233

120
13
100
150
270
400

Constructed Response:

433
67
+ 35

Performance Task:
Find a 3-digit number to subtract from 375 so that:

1. You don't have to use regrouping.
2. You would naturally use regrouping from the tens to the ones place.
3. You would naturally use regrouping from the hundreds place to the tens place.
4. You would naturally use regrouping in all places.

In each case, explain how you chose your numbers and complete the problem.

How will we respond when learning has not occurred?
How will we respond when learning has already occurred?

Curricular Connections

Math Concept Question:
Illustrate and explain multiplication and division using equations, rectangular array, and area models.

NBT.4 and NS.3.1; HM Math Unit 2 Chapter 3 Lesson 2,4,5, Unit 3 Chapter 4 Lesson 1,3,4.
NBT.5 and NS.3.2, NS.3.3; HM Math Unit 5 Chapter 10 Lesson 1.
NBT.6 and NS.3.2, NS.3.4; HM Math Unit 6 Chapter 12 Lesson 1-5, Unit 6 Chapter 13 Lesson 1-5.

Academic Vocabulary/Domain Specific Words
Place value, properties, sum, difference, product, quotient, remainder, digit, equations, rectangular array, area model, dividend, divisor, associative, commutative, identity, zero property of multiplication, algorithm

Skill Concepts: Illustrated and Explained

NBT.4
Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to fluency, which means accuracy and efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property, decomposing and recomposing numbers, etc.). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

\[
\begin{array}{c}
3892 \\
+1567
\end{array}
\]

Student explanation for this problem:
1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the 10 tens as one more hundred. (notates with a 1 above the hundreds column)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (notates with a 1 above the thousands column)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

\[
\begin{array}{c}
3546 \\
- 928 \\
\end{array}
\]

Student explanation for this problem:
1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.)
2. Sixteen ones minus 8 ones is 8 ones. (Writes an 8 in the ones column of answer.)
3. Three tens minus 2 tens is one ten. (Writes a 1 in the tens column of answer.)
4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (Marks through the 3 and notates with a 2 above it. (Writes down a 1 above the hundreds column.) Now I have 2 thousand and 15 hundreds.
5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (Writes a 6 in the hundreds column of the answer).
6. I have 2 thousands left since I did not have to take away any thousands. (Writes 2 in the thousands place of answer.)

Note: Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

NBT.5
Students who develop flexibility in breaking numbers apart (decomposing numbers) have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication and understanding why it works, is an expectation in the 5th grade.

This standard calls for students to multiply numbers using a variety of strategies.

<table>
<thead>
<tr>
<th>Student 2</th>
<th>Student 3</th>
<th>Student 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 x 12</td>
<td>25 x 12</td>
<td>25 x 12</td>
</tr>
<tr>
<td>I broke 25 up into 5 groups of 5</td>
<td>I doubled 25 and cut 12 in half to get 50 x 5</td>
<td>I broke 12 up into 10 and 2</td>
</tr>
<tr>
<td>5 x 12 = 60</td>
<td>12 x = 50</td>
<td>25 x 10 = 250</td>
</tr>
<tr>
<td>I have 5 groups of 5 in 25</td>
<td>50 x 6 = 300</td>
<td>25 x 2 = 50</td>
</tr>
<tr>
<td>60 x 5 = 300</td>
<td></td>
<td>250 + 50 = 300</td>
</tr>
</tbody>
</table>

Use of place value and the distributive property are applied in the scaffolded examples below.

- To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property, 154 X 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 x 6) + (4 x 6) = 600 + 300 + 24 = 924.
- The area model shows the partial products.
  14 x 16 = 224
Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

They use different strategies to record this type of thinking.

- Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

```
25
× 24
\underline{400} \quad (20 \times 20)
\underline{100} \quad (20 \times 5)
\underline{80} \quad (4 \times 20)
\underline{20} \quad (4 \times 5)
\underline{600}
```

```
25
× 24
\underline{500} \quad (20 \times 25)
\underline{100} \quad (4 \times 25)
\underline{600}
```

- **Matrix Model**: This model should be introduced after students have facility with the strategies shown above.
Example:
What would an array area model of 74 x 38 look like?

$$\begin{array}{c|c|c}
70 & 4 & 0 \\
\hline
30 & 70 \times 30 = 2100 & \text{ } \\
\text{ } & 70 \times 8 = 560 & 4 \times 30 = 120 \\
\text{ } & \text{ } & 4 \times 8 = 32 \\
\hline
\text{ } & \text{ } & 2000 = 560 + 1200 + 32 = 2812
\end{array}$$

**NBT.6**
In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

**Examples:**
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

**Using Base 10 Blocks:** Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.

**Using Place Value:** $260 \div 4 = (200 \div 4) + (60 \div 4)$

**Using Multiplication:** $4 \times 50 = 200, 4 \times 10 = 40, 4 \times 5 = 20; 50 + 10 + 5 = 65; \text{so } 260 \div 4 = 65$

This standard calls for students to explore division through various strategies.
Example:

Using an Open Array or Area Model
After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

150 ÷ 6

![Rectangle Diagram]

Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.
1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a fact or and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:
   a. 150 ÷ 6 = 10 + 10 + 5 = 25
      \[
      \begin{align*}
      \hline
      & \quad 6 \quad 6 \quad 6 \quad 6 \quad 6 \quad 6 \\
      60 & \quad 60 & \quad 60 & \quad 60 & \quad 60 & \quad 60 \\
      10 & \quad 10 & \quad 10 & \quad 10 & \quad 10 & \quad 10 \\
      1 & \quad 1 & \quad 1 & \quad 1 & \quad 1 & \quad 1 \\
      5 & \quad 5 & \quad 5 & \quad 5 & \quad 5 & \quad 5 \\
      \hline
      \end{align*}
      \]
   b. 150 ÷ 6 = (60 ÷ 6) + (60 ÷ 6) + (30 ÷ 6) = 10 + 10 + 5 = 25

Example 2:
1917 ÷ 9
A student’s description of his or her thinking may be:
I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines.
1917 ÷ 9 = 213
# Grade 4 Mathematics Instructional Unit 2: Segment 2

## What do we want all students to learn?

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<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>CCSS OA.4</td>
<td>'97 NS4.1</td>
<td>Mathematical Practices</td>
<td>Mathematical Practices</td>
</tr>
<tr>
<td>CCSS OA.5</td>
<td>'97 NS4.2*</td>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Reason abstractly and quantitatively</td>
<td>2. Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Use appropriate tools strategically</td>
<td>4. Use appropriate tools strategically</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5. Use appropriate tools strategically</td>
<td>5. Use appropriate tools strategically</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6. Attend to precision.</td>
<td>6. Attend to precision.</td>
</tr>
</tbody>
</table>

**Target(s):**

- B: Gain familiarity with factors and multiples.
- C: Generate and analyze patterns.

## Skills and Concepts

**When learning Concepts and Procedures students will . . .**

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

- **find all factor pairs for whole number in the range of 1-100.**
- **determine if a number is a multiple of a given one-digit number.**
- **determine if a number in the range of 1-100 is prime or composite.**

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting

**When problem solving students will . . .**

- **problem solve** by determining the factors and multiples of a number in order to gain familiarity with factors and multiples.

**While modeling and analyzing data students will . . .**

- **summarize** how they determined they generated a number or shape of a particular pattern or rule.

**While communicating and reasoning students will . . .**

- **generate a pattern that follows a given rule and identify features of the pattern that were not explicit in the rule.**
number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

- generate a number or shape pattern that follows a given rule.
- identify features in a pattern that were not explicit in a rule.

### Language Functions and Considerations

<table>
<thead>
<tr>
<th>Interpretation</th>
<th>Summarization</th>
<th>Inferring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phrases, sentences, or symbols to express understanding of the intended meaning of information. Domain specific vocabulary used.</td>
<td>Phrases or sentences to express relevant details about how they determined a particular pattern or rule. Domain specific vocabulary used.</td>
<td>Words, phrases, or sentences to express understanding of implied/implicit facts, based on available information. Domain specific vocabulary used.</td>
</tr>
<tr>
<td>Language Considerations: simple sentences, to be verbs (is/are), dependent clauses.</td>
<td>Language Considerations: Increasingly complex sentences with increasingly specific vocabulary.</td>
<td>Language Considerations: Connectors such as although, while, thus, and therefore.</td>
</tr>
<tr>
<td>The _____ of _____ is/are _____ because _____.</td>
<td>_____ is ________, and the result is ________.</td>
<td>_____ is ________, therefore the result is ________.</td>
</tr>
</tbody>
</table>

### How will we know if they have learned it?

**Performance Task and Formative Development:**
In 15 Instructional days students will be able to.......  

**Selected Response:**

Select the properties that apply to a prime number

- [ ] Can be divided evenly only by 2
- [ ] Can be divided evenly only by 1
- [ ] Can be divided evenly only by itself.
- [ ] Must be a whole number greater than 1
- [ ] Can be a whole number less than 1

**Mastery of Concepts and Skills: 16/20**
**Constructed Response:**

Identify the pattern and fill in the blank

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>200</td>
</tr>
</tbody>
</table>

**Performance Task:**

1. Make a list of the first ten multiples of 3.
2. Which of the numbers in your list are multiples of 6? What pattern do you see in where the multiples of 6 appear in the list?
3. Which numbers in the list are multiples of 7? Can you predict when multiples of 7 will appear in the list of multiples of 3? Explain your reasoning.

**How will we respond when learning has not occurred?**

**How will we respond when learning has already occurred?**

**Curricular Connections**

**Math Concept Question:**
Generate a number or shape pattern that follows a given rule and identify features of the pattern that were not explicit in the rule.

Find the difference between a prime number and a composite number.

OA.4 and NS4.1, NS4.2; HM Math Unit 5 Chapter 11 Lesson 1, Unit 6 Chapter 12 Lesson 4, Unit 6 Chapter 14 Lesson 1, 3, 4.
OA.5 HM Math Lesson Unit 3 Chapter 6 Lesson 4, Unit 4 Chapter 9 Lesson 1-4, Unit 6 Chapter 14 Lesson 2.
OA.4
This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Prime vs. Composite:
A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors.

Students investigate whether numbers are prime or composite by
• building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)
• finding factors of the number

Students should understand the process of finding factor pairs so they can do this for any number 1 – 100.

Example:
Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:
Factors of 24: 1, 2, 3, 4, 6, 8,12, 24
Multiples: 1, 2, 3, 4, 5...24
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
3, 6, 9, 12, 15, 18, 21, 24
4, 8, 12, 16, 20, 24
8, 16, 24
12, 24
24

To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints include the following:
• all even numbers are multiples of 2
• all even numbers that can be halved twice (with a whole number result) are multiples of 4
• all numbers ending in 0 or 5 are multiples of 5

OA.5
Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Examples:

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Rule</th>
<th>Feature(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 8, 13, 18, 23, 28, ...</td>
<td>Start with 3, add 5</td>
<td>The numbers alternately end with a 3 or 8</td>
</tr>
<tr>
<td>5, 10, 15, 20, ...</td>
<td>Start with 5, add 5</td>
<td>The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.</td>
</tr>
</tbody>
</table>

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:
Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18, etc.)

In this standard, students describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

<table>
<thead>
<tr>
<th>Day</th>
<th>Operation</th>
<th>Beans</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3x0+4</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>3x1+4</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>3x2+4</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3x3+4</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>3x4+4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>3x5+4</td>
<td>19</td>
</tr>
</tbody>
</table>
Grade 4 Mathematics Instructional Unit 2: Segment 3

Unit Big Idea: Change
Suggested Duration: 15 Days

Segment Idea: Changes in fraction form

What do we want all students to learn?

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS</td>
<td>'97</td>
<td>Mathematical Practices</td>
<td>Mathematical Practices</td>
</tr>
<tr>
<td>NF.1</td>
<td>NS1.2*</td>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>NF.2</td>
<td>NS1.6</td>
<td>7. Look for and make use of structure.</td>
<td>2. Reason abstractly and quantitatively</td>
</tr>
<tr>
<td>NF.5</td>
<td>NS1.7</td>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>5. Use appropriate tools strategically.</td>
</tr>
<tr>
<td>NF.6</td>
<td>NS1.9*</td>
<td>5. Use appropriate tools strategically.</td>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
</tr>
</tbody>
</table>

Target(s):
F: Extend understanding of fraction equivalence and ordering.
H: Understand decimal notation for fractions, and compare decimal fractions.

Skills and Concepts

When learning Concepts and Procedures students will . . .

Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (NF.1)

- write equivalent fractions using visual models.
- explain why two or more fractions are equivalent.

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a

When problem solving students will . . .

- explain how to express equivalent fractions with a denominator of 10 and 100, as well as convert fractions to decimals, and compare fractions and decimals.

While modeling and analyzing data students will . . .

- model how to compare fractions and decimals.

While communicating and reasoning students will . . .

- write equivalent fractions for a given fraction and justify why the fractions are equivalent.
benchmark fraction such as 1/2.
Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (NF.2)

- compare two different fractions by creating common denominators.
- compare two different fractions by comparing to a benchmark fraction such as 1/2.
- justify which fraction is larger or smaller by using a visual fraction model.

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

- express a fraction with denominator of 10 with a denominator of 100.
- add two fractions with respective denominators of 10 and 100.

Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. (NF.6)

- use decimal notation for fractions with denominators 10 or 100.

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the
symbols >, =, or <, and justify the conclusions, e.g., by using the number line or another visual model. (CA) (NF.7)

- compare decimals to hundredths using < or =.

### Language Functions and Considerations

<table>
<thead>
<tr>
<th>Explanation</th>
<th>Compare</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Explanation&quot;—Sentences to express relationships between two or more processes. Domain specific vocabulary used.</td>
<td>&quot;Compare&quot;—Expressing similarities. Domain specific vocabulary used.</td>
<td>&quot;Description&quot;—Sentences to express or observe the attributes or properties of two fractions. Domain specific vocabulary used.</td>
</tr>
<tr>
<td>Language Considerations: verb forms, indicative verbs, declarative sentences and complex sentences. Adverbs of manner, therefore, as a result, for that reason.</td>
<td>Language Considerations: adjectives and conjunctions, comparatives.</td>
<td>Language Considerations: simple sentences, to be verbs (is/are), dependent clauses.</td>
</tr>
<tr>
<td>The _____ of _____ is ____, therefore ______________.</td>
<td>_____ and _____ are alike because they both __________.</td>
<td>_____ equals ____, because ______________.</td>
</tr>
</tbody>
</table>

### How will we know if they have learned it?

**Performance Task and Formative Development:**
In 15 Instructional days students will be able to........

**Mastery of Concepts and Skills: 16/20**

**Selected Response:**
Select all statements that are true about the following fraction

- □ The blue and red fractions are equal
- □ The red fraction is 2/8
- □ The number of parts in the red and blue fractions are equal
- □ The blue fraction is 6/8
The size of the parts in the blue fraction are not equal
The size of the parts in the red fraction are equal

**Constructed Response:**

Solve

\[
\begin{array}{cc}
88 & + \\
& 20 \\
\hline
& \\
10 & 100
\end{array}
\]

**Performance Task:**

Jessica and some friends have ordered two pizzas. One is a medium sized pizza while the other is a large.

Jessica eats two slices of the medium sized pizza. Has Jessica eaten \( \frac{2}{6} \) of the two pizzas? Explain your reasoning, and draw a picture to illustrate your explanation.
How will we respond when learning has not occurred?

How will we respond when learning has already occurred?

Curricular Connections

Math Concept Question:
Explain why two fractions are equivalent.
Explain how to change a fraction with a denominator of 10 to a denominator of 100.

NF.1; HM Math Unit 8 Chapter 17 Lesson 3-4.
NF.2; HM Math Unit 8 Chapter 18 Lesson 1,2,4.
NF.5 and NS1.7; HM Math Unit 9 Chapter 19 Lesson 4-6.
NF.6 and NS1.6; HM Math Unit 9 Chapter 19 Lesson 4-6.
NF.7 and NS1.7, NS1.9, NS1.2; HM Math Unit 9 Chapter 19 Lesson 4-5.

Academic Vocabulary/Domain Specific Words
Fraction, equivalent, model, numerator, denominator, compare, common denominator, decimal notation, justify, tenths, hundredths, halves, fourths, number line

Skill Concepts: Illustrated and Explained

NF.1
This standard refers to visual fraction models. This includes area models, linear models (number lines) or it could be a collection/set models.

This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).

Students can use visual models or applets to generate equivalent fractions.

Example:
All the area models show 1/2. The second model shows 2/4 but also shows that 1/2 and 2/4 are equivalent fractions because their areas are equivalent. When a horizontal line is drawn through the center of the model, the number of equal parts doubles and size of the parts is halved.

Students will begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions.
\[ \frac{1}{2} \times \frac{2}{2} = \frac{2}{4}. \]

Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80

**NF.2**

This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. **Students’ experiences should focus on visual fraction models rather than algorithms.** When tested, models may or may not be included. Students should learn to draw fraction models to help them compare and use reasoning skills based on fraction benchmarks. Students must also recognize that they must consider the size of the whole when comparing fractions (i.e., 1/2 and 1/8 of two medium pizzas is very different from 1/2 of one medium and 1/8 of one large). Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

**Example:**

Use patterns blocks.
1. If a red trapezoid is one whole, which block shows 1/3?
2. If the blue rhombus is 1/3, which block shows one whole?
3. If the red trapezoid is one whole, which block shows 2/3?

Melisa used a 12 x 12 grid to represent 1 and Nancy used a 10 x 10 grid to represent 1. Each girl shaded grid squares to show \(\frac{1}{4}\). How many grid squares did Melisa shade? How many grid squares did Nancy shade? Why did they need to shade different numbers of grid squares?

Possible solution: Melisa shaded 36 grid squares; Nancy shaded 25 grid squares. The total number of little squares is different in the two grids, so 1/3 of each total number is different.
Example:
There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 left. Which cake has more left?

Student 1
Area model:
The first cake has more left over. The second cake has 5/12 left which is smaller than ½.

Student 2
Linear/Number Line model:
Student 3:
I know that 6/12 equals 1/2. Therefore, the second cake which has 7/12 left is greater than 1/2. **Benchmark fractions** include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths.

Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include <, >, =.

**It is important that students explain the relationship between the numerator and the denominator, using Benchmark Fractions. See examples below:**

Fractions may be compared using $\frac{1}{2}$ as a benchmark.

**NF.5**

This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work. **Student experiences should focus on working with grids rather than algorithms.** Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.
Example:
Represent 3 tenths and 30 hundredths on the models shown below:

<table>
<thead>
<tr>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tenths Grid</td>
<td>.3 = 3 tenths = 3/10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hundredths Grid</td>
<td>.30 = 30 hundredths = 30/100</td>
<td></td>
<td></td>
</tr>
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</table>

10ths circle  
100ths circles

Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

*Base Ten Blocks:* students may represent 3/10 with 3 longs and may also write the fraction as 30/100 with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart as shown in 4.NF.6.
Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

<table>
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<tr>
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<td></td>
<td>.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Students use the representations explored in 4.NF.5 to understand $\frac{32}{100}$ can be expanded to $\frac{3}{10}$ and $\frac{2}{100}$.

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.

![Number Line](image)

**NF.6**

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

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![Number Line](image)
NF.7
Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows $\frac{3}{10}$ but the whole on the right is much bigger than the whole on the left. They are both $\frac{3}{10}$ but the model on the right is a much larger quantity than the model on the left.

When the wholes are the same, the decimals or fractions can be compared.

Example:
Draw a model to show that $0.3 < 0.5$. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.)